Statistics 101 - Take Home Lab #10B

In order to be able to do statistical inference for the population mean μ, we needed to introduce a new distribution. The t distribution allows us to calculate confidence intervals and perform hypothesis testing for the population mean μ. But why is it so important that we learn this new distribution? What would happen if we just stuck with the normal distribution (z) instead?

To look for answers to these questions, go to the course webpage at

http://streaming.stat.iastate.edu/~stat101/homepage.html

Under **Computing**, click on the link **Confidence Interval for a Mean**. The applet will take a few moments to load. Once the applet is ready, we would like to look at confidence intervals for the mean. Under Method: choose **Means** in the first drop-down box and choose **t** in the second drop-down box. We will simulate confidence intervals for the mean height of a population of males so enter $\mu = 69$ (inches) and $\sigma = 3$ (inches). Our samples will have size 5, so enter $n = 5$.

1. Press the button **Sample**. The applet will generate one sample of size 5 and calculate the confidence interval for μ using the formula

$$\overline{y} \pm t^* \left(\frac{s}{\sqrt{n}} \right)$$

   When you click on the confidence interval in the graph, the end-points of the confidence interval are given along with the sample mean and sample standard deviation. Write down your confidence interval, $\overline{y}$ and s.

2. Under Method: change the second drop-down box to **z with s**. Your confidence interval from #1 has changed. The formula used to calculate this confidence interval is

$$\overline{y} \pm z^* \left(\frac{s}{\sqrt{n}} \right)$$

   Click on the confidence interval in the graph and record the end-points of the confidence interval.

3. Describe the difference between the confidence interval calculated in #1 and the confidence interval calculated in #2.

4. Under Method: change the second drop-down box to **t**. Just above the **Sample** button, change the number of intervals to **100**. Now press the **Sample** button. You should now have 100 confidence intervals in the graph with each confidence interval calculated using the formula in #1. What proportion of your 100 confidence intervals contain the mean μ?

5. Click the **Sample** button 9 more times. Under the **Running Total**, you should see results for 1000 confidence intervals. What proportion of these 1000 confidence intervals contain the mean μ? What value should your proportion be close to?

6. At the bottom of the applet, click the button **Reset**. Under Method: change the second drop-down box to **z with s**. Now press the **Sample** button. You should now have 100 confidence intervals in the graph with each confidence interval calculated using the formula in #2. What proportion of your 100 confidence intervals contain the mean μ?

7. Click the **Sample** button 9 more times. Under the **Running Total**, you should see results for 1000 confidence intervals. What proportion of these 1000 confidence intervals contain the mean μ? What value should your proportion be close to?

8. Which method gives you a coverage rate (proportion of confidence intervals containing μ) closer to the confidence level: the **t** method or the **z with s** method?

9. Based on what you have seen so far, fill in the blanks: When calculating a confidence interval for a population _____, we need to use the _____ distribution so that the _____ rate for our confidence interval will be close to the confidence _____ of the interval.