1. Examine the estimated correlation matrix, R for the study of chicken-bone measurements described in example 9.14 in the Johnson and Wichern. The correlation matrix is given in problem 9.10 in Johnson and Wichern along with maximum likelihood estimates of factor loadings for a two factor model. Varimax rotated estimated factor loadings are also given. These are shown in the following table.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum Likelihood Estimates of Factor Loadings</th>
<th>Varimax Rotation of Estimated Factor Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F_1</td>
<td>F_2</td>
</tr>
<tr>
<td>Skull length</td>
<td>.602</td>
<td>.200</td>
</tr>
<tr>
<td>Skull breadth</td>
<td>.467</td>
<td>.154</td>
</tr>
<tr>
<td>Femur length</td>
<td>.926</td>
<td>.143</td>
</tr>
<tr>
<td>Tibia length</td>
<td>1.000</td>
<td>.000</td>
</tr>
<tr>
<td>Humerus length</td>
<td>.874</td>
<td>.476</td>
</tr>
<tr>
<td>Ulna length</td>
<td>.894</td>
<td>.327</td>
</tr>
</tbody>
</table>

a. Using the estimated loadings of the six variables on the varimax rotated factors, compute the communalities and specific variances.

specific variances: \( .597 \), \( .758 \), \( .122 \), \( .000 \), \( .010 \), \( .094 \)

communalities: \( .403 \), \( .242 \), \( .878 \), \( 1.000 \), \( .990 \), \( .906 \)

Note that the factor loadings correspond to a Heywood case in which all of the variation in tibia lengths is explained by variation among chickens in the values for the two factors.

b. Report the proportion of total standardized sample variation accounted for by each of the two rotated factors, and give a one or two sentence interpretation of each rotated factor.

Factor 1 is an overall size component that emphasizes wing size measurements. It accounts for 38.4% of the variation in the standardized traits.

Factor 2 is an overall size component that emphasizes leg measurements. It accounts for 35.3% of the variation in the standardized traits.

c. How do loadings for the rotated factors correspond to patterns in the correlation matrix?

All traits have moderate to large positive correlations. Consequently, all of the traits have positive loadings on the first two factors. Since the wing measurements \( (X_5 \text{ and } X_6) \) have the strongest correlation, they are featured in the first factor. The leg measurements \( (X_3 \text{ and } X_4) \) are almost as strongly correlated as the wing measurements, and they are featured in the second factor.
d. The residual matrix \( R - \hat{L}_z \hat{L}_z - \hat{\psi}_z \) is shown below. Only the correlation between the two skull measurements, which was the weakest positive correlation, is not well explained by the loadings on the two factors that emphasize variation and correlation among the measurements of lengths of leg bones and wing bones. This occurs because there are stronger positive correlations within and between wing bones and leg bones measurements than the correlations between skull and wing measurements and the correlations between skull and leg measurements (examine the original correlation matrix).

\[
\begin{bmatrix}
0 & 0.1924 & -0.0175 & -0.0006 & -0.0008 & -0.0012 \\
0.1924 & 0 & -0.0328 & -0.0004 & -0.0001 & -0.0185 \\
-0.0175 & -0.0328 & 0 & 0.0000 & 0.0000 & 0.0035 \\
-0.0006 & -0.0004 & 0.0000 & 0 & 0.0000 & -0.0000 \\
-0.0008 & -0.0001 & 0.0000 & 0.0000 & 0 & 0.0000 \\
-0.0012 & -0.0185 & 0.0035 & -0.0000 & 0.0000 & 0 \\
\end{bmatrix}
\]

e. How would your answers to parts b and d change if you used the estimated loadings for the un-rotated factors instead of the estimated loadings for the varimax rotated factors?

The combined amount of variation accounted for by two factors would be the same for a two factor model using either rotated or unrotated factors. Of course, the rotated factors do not have the same interpretation as the unrotated factors. The first unrotated factor is an overall size factor with greater emphasis on wing and leg measurements than on skull measurements. It accounts for about 67 percent of the variation in the standardized traits. The second unrotated factor is a wing size factor that accounts for deviations of wing sizes from what is predicted by the overall size factor. This reflects that correlations between wing and leg measurements are smaller than correlations between leg measurements and also smaller than correlations between wing measurements. This factor accounts for about 7 percent of the variation in the standardized traits. The communalities and specific variances, reported in part (a), are the same for two rotated factors and the two unrotated factors. The residual matrix in part (d) should be the same for both sets of factors, and any differences in computer output are results of rounding in the storage of numbers.

f. A plot of the loadings for the varimax rotated factors is shown on the next page. This plot indicates that orthogonal factors are not really possible for the leg bones and the wing bones. Non-orthogonal axes are drawn on the plot to show that a PROMAX transformation could produce two factors for which leg bone lengths load highly on one factor and wing measurements would have relatively small loadings on that factor. The wing bone measurements would load highly on the second factor and leg bone measures would have relatively small loadings on the second factor. Skull measurements would have moderate loadings on each factor. Loadings on the PROMAX factors are displayed below the plot. Those factors have correlation 0.71.
Loadings on PROMAX transformed factors as indicated in the previous graph.

<table>
<thead>
<tr>
<th>Rotated Factor Pattern (Standardized Regression Coefficients)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>X1</td>
</tr>
<tr>
<td>X2</td>
</tr>
<tr>
<td>X3</td>
</tr>
<tr>
<td>X4</td>
</tr>
<tr>
<td>X5</td>
</tr>
<tr>
<td>X6</td>
</tr>
</tbody>
</table>
2. (a) \[
\begin{bmatrix}
1 & \rho & \rho & \rho \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
\rho & \rho & \rho & 1
\end{bmatrix}
\begin{bmatrix}
0.5 \\
0.5 \\
0.5 \\
0.5
\end{bmatrix}
= (3\rho + 1)
\begin{bmatrix}
0.5 \\
0.5 \\
0.5 \\
0.5
\end{bmatrix}
\quad \text{and} \quad \hat{\lambda}_1 = 3\rho + 1
\]

(b) \[
\hat{\lambda}_2 = \hat{\lambda}_3 = \hat{\lambda}_4 = 1 - \rho
\]

One set of eigenvectors is
\[
\begin{bmatrix}
1/\sqrt{2} \\
-1/\sqrt{2} \\
0 \\
0
\end{bmatrix}
\quad \text{and} \quad \begin{bmatrix}
1/\sqrt{6} \\
1/\sqrt{6} \\
-2/\sqrt{6} \\
0
\end{bmatrix}
\]

This eigenvectors provide a basis for a three dimensional subspace that is orthogonal to the first eigenvector from part (a). Since the eigenvalues are equal, Any rotation of these three eigenvectors within that three dimensional subspace also yields an allowable set of eigenvectors.

(c) Defining the vector of loadings and the diagonal matrix of specific variances as
\[
L = \begin{bmatrix}
\sqrt{\rho} \\
\sqrt{\rho} \\
\sqrt{\rho} \\
\sqrt{\rho}
\end{bmatrix}
\quad \text{and} \quad \Psi = (1 - \rho)I,
\]

we have
\[
\begin{bmatrix}
1 & \rho & \rho & \rho \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
\rho & \rho & \rho & 1
\end{bmatrix}
= LL' + \Psi
\]

(d) No, the correlations will not be equal unless each trait has the same loading on the single factor. An example of a correlation matrix with unequal correlations that satisfies a one factor model is
\[
R = \begin{bmatrix}
1 & .125 & .125 & .125 \\
.125 & 1 & .250 & .250 \\
.125 & .250 & 1 & .250 \\
.125 & .250 & .250 & 1
\end{bmatrix}
= \begin{bmatrix}
.25 \\
.50 \\
.50 \\
.50
\end{bmatrix}
\begin{bmatrix}
.9375 & 0 & 0 & 0 \\
0 & .75 & 0 & 0 \\
0 & 0 & .75 & 0 \\
0 & 0 & 0 & .75
\end{bmatrix} + \begin{bmatrix}
.0000 \\
.0000 \\
.0000 \\
.0000
\end{bmatrix}
\]

3. (a) Assume the model \( X_i = a_i F + \varepsilon_i \) for \( i = 1, 2, \ldots, p \) traits, where
\[
E(F) = \mu_F \quad \text{Var}(F) = \sigma_F^2 \quad E(\varepsilon_i) = 0 \quad \text{Var}(\varepsilon_i) = \psi_i
\]

\[
\text{Cov}(\varepsilon_i, \varepsilon_j) = 0 \quad \text{and} \quad \text{Cov}(\varepsilon_i, F) = 0 \quad \text{for all} \ i, j = 1, 2, \ldots, p.
\]
Then
\[ X = \begin{bmatrix} X_1 \\ \vdots \\ X_p \end{bmatrix} = F + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_p \end{bmatrix} \]

where
\[ E \begin{bmatrix} \varepsilon_i \\ \vdots \\ \varepsilon_p \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{and} \quad V \begin{bmatrix} \varepsilon_i \\ \vdots \\ \varepsilon_p \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_p \end{bmatrix} \]

and
\[ \text{Var}(X_i) = \text{Var}(a_i F + \varepsilon_i) = a_i^2 \sigma_F^2 + \psi_i, \quad \text{for } i = 1, \ldots, p \]
\[ \text{Cov}(X_i, X_j) = \text{Cov}(a_i F + \varepsilon_i, a_j F + \varepsilon_j) = a_i a_j \sigma_F^2, \quad \text{for all } i \neq j \]
\[ \text{Corr}(X_i, X_j) = \rho_{ij} = \frac{\text{Cov}(X_i, X_j)}{\sqrt{\text{Var}(X_i)} \sqrt{\text{Var}(X_j)}} = \frac{a_i a_j \sigma_F^2}{\sqrt{a_i^2 \sigma_F^2 + \psi_i} \sqrt{a_j^2 \sigma_F^2 + \psi_j}} \]
\[ = a_i a_j \quad \text{if } \sigma_F^2 = 1. \]

Then the ratio of correlations is
\[ \frac{\rho_{ik}}{\rho_{jk}} = \frac{\frac{a_i a_k \sigma_F^2}{\sqrt{a_i^2 \sigma_F^2 + \psi_i} \sqrt{a_k^2 \sigma_F^2 + \psi_k}}}{\frac{a_j a_k \sigma_F^2}{\sqrt{a_j^2 \sigma_F^2 + \psi_j} \sqrt{a_k^2 \sigma_F^2 + \psi_k}}} = \frac{a_i a_k \sigma_F^2}{\sqrt{a_i^2 \sigma_F^2 + \psi_i}} \frac{\sqrt{a_j^2 \sigma_F^2 + \psi_j}}{a_j a_k \sigma_F^2} = \frac{a_i \sqrt{a_i^2 \sigma_F^2 + \psi_i}}{a_j \sqrt{a_j^2 \sigma_F^2 + \psi_j}} \]

remains constant as you move across k, as long as i \neq k and j \neq k.

(b)

Since
\[ \text{Var}(X_i) = \text{Var}(a_i F + \varepsilon_i) = a_i^2 \sigma_F^2 + \psi_i, \] we have
\[ a_i^2 = \frac{\text{Var}(X_i) - \psi_i}{\sigma_F^2} \]

Since the variables are standardized we have \( \text{Var}(X_i) = 1 \), and taking \( \sigma_F^2 = 1 \) yields
\[ a_i^2 = 1 - \psi_i, \quad \text{for } i = 1, \ldots, p. \]
(c) Compute $r_k = \frac{1}{p-1} \sum_{i=k}^p r_{ik}$, for $k=1,\ldots,p$, and $\bar{r} = \frac{2}{p(p-1)} \sum_{i<j} r_{ij}$

$$\hat{\gamma} = \frac{(p-1)^2 (1-(1-\bar{r})^2)}{p(p-2)(1-\bar{r})^2} \quad \text{and} \quad T = \frac{n-1}{(1-\bar{r})^2} \left[ \sum_{i<j} (r_{ij} - \bar{r})^2 - \hat{\gamma} \sum_{k=1}^p (r_k - \bar{r})^2 \right] = 16.7$$

with $(p+1)(p-2)/2 = 14$ d.f. and p-value $= .272$.

The hypothesis of equal correlations is not rejected.

In this case, $n=33$, $p=6$, $\bar{r}_1 = 0.72$, $\bar{r}_2 = 0.678$, $\bar{r}_3 = 0.628$,
$\bar{r}_4 = 0.594$, $\bar{r}_5 = 0.540$, $\bar{r}_6 = 0.524$, $\bar{r} = 0.614$ and $\hat{\gamma} = 3.937$.

Here is an R function to evaluate the Lawley test:

```r
Lawley.test<-function(n, corr)
{p<-nrow(corr)
r.kbar<-rep(0,p)
for(i in 1:p){r.kbar[i]<-(1/(p-1))*sum(corr[i,])-corr[i,i])}
r.bar<-mean(r.kbar)
gamma<-(p-1)^2*(1-(1-r.bar)^2)/(p-(p-2)*(1-r.bar)^2)
corr.diff<-(corr-r.bar)^2
corr.diff.rsum<-rep(0,p-1)
for(i in 1:(p-1)){corr.diff.rsum[i]<-sum(corr.diff[i,(i+1):p])}
corr.diff.sum<-sum(corr.diff.rsum)
T<-(n-1)/(1-r.bar)^2)*(corr.diff.sum-gamma*sum((r.kbar-r.bar)^2))
df<-(p+1)*(p-2)/2
p.val<-1-pchisq(T,df)
return(list(T,df,p.val))}
```

For our data, we obtain the following result:

```r
Lawley.test(33, spcorr)
[[1]]
[1] 16.71036
[[2]]
[1] 14
[[3]]
[1] 0.2719414
```
4. (i) A one factor model appears to be adequate. In this case, the value of the chi-square test statistic is 2.516 on 9 d.f., with p-value = 0.989.

(ii) This is consistent with the result in part (c) of problem 3. As shown in problem 2, a one factor model is adequate when all correlations are equal.

5. The data for this example consist of scores for 24 psychological tests given to 145 seventh and eighth grade school children in a suburb of Chicago. These data were collected by Karl Holzingen and Frances Swineford (1937, Psych. 2, 42-54) and they have become a classic example in the factor analysis literature.

a. Principal component analysis of the correlation matrix using a varimax rotation of the first 5 components and yielded the following five rotated factors:

   Factor 1: Reading and verbal ability factor with some emphasis on problem reasoning. Perhaps subjects with better verbal skills are better able to understand problems and express ideas for solving problems.

   Factor 2: Perceptual speed / mathematical ability factor

   Factor 3: Spatial relation and visual perception ability with some emphasis on deductive reasoning and series completion.

   Factor 4: Recognition ability. Subjects with higher values of this factor are better able to recognize words, numbers, figures and object-number associations.

   Factor 5: Subjects with higher values for this factor have greater ability to memorize figure-word associations.

b. A varimax rotation of the five factors obtained from a factor analysis with the iterated principal factor method yielded five factors that are similar to those obtained from rotating principal components in part (a). There are some small differences.

   Factor 1: Reading and verbal ability factor.

   Factor 2: Spatial relation and visual perception ability with some emphasis on deductive reasoning and problem solving.

   Factor 3: Perceptual speed / mathematical ability factor.

   Factor 4: Word and symbol recognition and ability to memorize object-number and number-figure associations.
Factor 5: Ability to memorize figure-word associations and perceptual speed.

c. Obtain maximum likelihood estimates for factor loadings for the first five factors and perform a varimax rotation. Make comparisons with the results from parts a and b.

Varimax rotation of five factors obtained from maximum likelihood estimation produces factors similar to those reported in part (b), but these factors may be a little easier to interpret. The third factor, for example, is more clearly a perceptual speed and mathematical ability factor. The fourth factor now combines word, number, and figure recognition with the ability to memorize object-number, number-figure, and figure-word associations.

d. Determine how many factors are needed. Give some justification for your answer. If your answer is not 5 factors, compute maximum likelihood estimates for factor loadings for the appropriate number of factors, do a varimax rotation, and give interpretations of the rotated factors.

Both the SCREE plot and the chi-square tests based on maximum likelihood estimation indicate that 5 factors are adequate and 4 factors provide a solution that is almost as good. Results for the chi-square tests are:

- Ho: Five factors are sufficient. $X^2 = 186.8$ on 166 d.f. with p-value = 0.128
- Ho: Four factors are sufficient. $X^2 = 226.7$ on 186 d.f. with p-value = 0.022

The fifth rotated factor explains only about 2% of the variation in the standardized scores, and many people found it difficult to interpret. The AIC criterion decreases only slightly when a fifth factor is included, while the Schwarz criterion increases. It seems reasonable to use four factors, but only five factor solutions are shown below because the varimax rotation of the four factor solution essentially yields the first four factors obtained from rotating the five factor solution.

e. The PROMAX transformation produces factors similar to those obtained from the VARIMAX rotation, but they are more sharply focused on separate subsets of variables. These factors have moderate positive correlations of about 0.4 with each other.

f. Unlike the varimax rotation and the promax transformation (which make use of a varimax rotation in an intermediate step), the quartimax rotation does not destroy the overall performance factor. Then it proceeds to create additional factors for which different subsets of variables have substantial loadings on only one of the additional factors.
Factor 1: Overall intelligence or general performance factor.

Factor 2: Verbal ability and comprehension factor.

Factor 3: Perceptual speed / mathematical ability factor

Factor 4: Symbol recognition and ability to memorize object-number and number-figure associations

Factor 5: Object-number and figure-word association and geometry skills.

Here is some R code for doing computations associated with the above analysis.

```r
# This code uses R to perform a factor analysis on
# the test scores data for problem 5 on assignment 5.
# The data are posted as test2.cor. The sample size is n=145.

tdata <- read.table("c:/documents and settings/kkoehler.IASTATE/my documents/courses/st501/data/test2.cor", header=F, col.names=c("x1", "x2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10", "x11", "x12", "x13", "x14", "x15", "x16", "x17", "x18", "x19", "x20", "x21", "x22", "x23", "x24"))

# Extract the sample sizes, means, standard deviations and the correlation matrix
n<-tdata[1,1]
tmeans<-tdata[2,]
tstd<-tdata[3,]

tcorr <- as.matrix(tdata[4:27,])
dimnames(tcorr) <- list(c("x1", "x2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10", "x11", "x12", "x13", "x14", "x15", "x16", "x17", "x18", "x19", "x20", "x21", "x22", "x23", "x24"), c("x1", "x2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10", "x11", "x12", "x13", "x14", "x15", "x16", "x17", "x18", "x19", "x20", "x21", "x22", "x23", "x24"))
tcorr

# Compute factors corresponding to principal components
# Note that rotation="none" is used to prevent the
# default application of a varimax rotation.

tfacp <- factanal(covmat=tcorr, factors=3, n.obs=n, method='prin', rotation="none")
tfacp

# Compute the residual matrix
```
pred <- tfacp$loadings %*% t(tfacp$loadings) + diag(tfacp$uniqueness)
resid <- tcorr - pred
resid

# Compute factors corresponding to principal components
# Note that rotation = "varimax" is used to apply a varimax
# rotation. This also occurs when you do not indicate a rotation.

tfacp <- factanal(covmat = tcorr, factors = 3, n.obs = n,
                  method = 'prin', rotation = "none")
tfacp

# Compute the residual matrix
pred <- tfacp$loadings %*% t(tfacp$loadings) + diag(tfacp$uniqueness)
resid <- tcorr - pred
resid

# Compute factors corresponding to the maximum likelihood estimation method

tfacmle <- factanal(covmat = tcorr, factors = 3, n.obs = n, method = 'mle')
tfacmle

# Compute the residual matrix
pred <- tfacmle$loadings %*% t(tfacmle$loadings) + diag(tfacmle$uniqueness)
resid <- tcorr - pred
resid

# To do other rotations we must download the the packages called
# psych and GPArotation and attach the libraries

library(psych)
library(GPArotation)

# You can get information on the Psych package and the fa( )
# function to perform factor analysis by typing help('fa') or help('psych').

# Here is an example of the fa function using a few options
# r = name of the correlation matrix
# nfactors = number of factors to use
# rotate = type of rotation
# n.obs = number of subjects in the data set
# (This is needed to get a likelihood ratio test of the null hypothesis that
# the specified number of factors is adequate)
# residuals = TRUE computes the residual matrix
# max.iter = 50 sets the maximum number of iterations
# to find the mle for the loadings equal to 50
#
# use maximum likelihood estimation with a varimax rotation

```r
famle <- fa(r=tcorr, nfactors=5, rotate="varimax", n.obs=145,
            scores="regression", max.iter=50)
```

# print some results

```r
famle
```

# Print the residual matrix

```r
famle$residual
```

# Print the test for an appropriate number of factors

```r
famle$STATISTIC
famle$PVAL
```

# use maximum likelihood estimation with a quartimax rotation

```r
famle <- fa(r=tcorr, nfactors=5, rotate="quartimax", n.obs=145
            scores="regression", max.iter=50, residuals=TRUE)
```

famle

```r
summary(famle)
```

# Print the residual matrix

```r
famle$residual
famle$STATISTIC
famle$PVAL
```

6. Suppose that n1 = 11 and n2 = 21 observations are sampled from two different bivariate normal distributions which have a common covariance matrix and possibly different mean vectors. The sample mean vectors and pooled covariance matrix are:

\[
\begin{align*}
\bar{X}_1 &= \begin{bmatrix} -1 \\ -2 \end{bmatrix}, & \bar{X}_2 &= \begin{bmatrix} 4 \\ 5 \end{bmatrix}, & S &= \begin{bmatrix} 13 & 6 \\ 6 & 22 \end{bmatrix}
\end{align*}
\]

a. The following R Code was used to perform the Hotelling two-sample $T^2$ statistic to test the null hypothesis that the population mean vectors are equal.
Spooled <- matrix(c(13, 6, 6, 22), ncol = 2)
Spooled

[,1] [,2]
[1,] 13 6
[2,] 6 22

xbar1 <- c(-1, -2)
xbar2 <- c(4, 5)
n1 <- 11
n2 <- 21
p <- 2
T2 <- (n1 * n2/(n1 + n2)) * t(xbar1 - xbar2) %% solve(Spooled) %% (xbar1-xbar2)

F = (n1 + n2 - p - 1)/((n1 + n2 - 2) * p) * T2
df1 <- p
df2 <- n1 + n2 - p - 1

T2

[,1]
[1,] 22.14713

F

[,1]
[1,] 10.70444
df1

[1] 2
df2

[1] 29

1 - pf(F, df1, df2)

[,1]
[1,] 0.000329927

So $T^2 = 22.15$ and $F = 10.71$ with $(2, 29)$ degrees of freedom and a $p$-value of 0.00033. Since $p = 0.00033 < 0.05$, we reject the null hypothesis and conclude that the evidence strongly suggests that the two population mean vectors are different.

b. An estimate of the classification rule based on Fisher’s linear discriminant function (formula 11-35 in the text) is obtained as follows:

$$\hat{m} = 0.5(\overline{x}_1 - \overline{x}_2)'S^{-1}_{pooled}(\overline{x}_1 + \overline{x}_2)' = -0.774$$

and

$$\hat{y}_0 = (\overline{x}_1 - \overline{x}_2)'S^{-1}_{pooled}x_0 = -0.272x_{0,1} - 0.244x_{0,2}$$
Classify a case with measurements \( x_0 \) into population 1 if \( 0.774 - 0.272x_{0,1} - 0.244x_{0,2} > 0 \) and classify the case into population 2 if \( 0.774 - 0.272x_{0,1} - 0.244x_{0,2} > 0 \).

c. Consider an observation \( x_0 = [2, 1]' \) on a new experimental unit. Calculate \( 0.774 - (0.272)(2) - (0.244x_{0,2})(1) = -0.014 \). We would classify this case into population 2.

d. Classify the unit in part (c) assuming prior probabilities .35 and .65 of observing a unit from population 1 and 2, respectively. Also, assume the cost of misclassifying a unit from population 2 into population 1 is ten times greater than the cost of misclassifying a unit from population 1 into population 2. Using the Estimated Minimum ECM Rule for two normal populations (11-18 in the textbook), we classify the case into population 1 because

\[
0.774 - (0.272)(2) - (0.244x_{0,2})(1) = -0.014 > \log \left( \frac{(l)(0.65)}{(10)(0.35)} \right) = -1.68
\]